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NOTE: The fact that such points are possible in individual cases is indicated, for example, by the analysis of the behavior of magnetization curves for monocrystals with positive anisotropy during magnetization parallel to the diagonal of a cube, the so-called "Gerlach's peaks."

Consequently, relations (1) and (2) can be generalized on the basis of the Taylor-Laurent formulas, to give:

$$\frac{dI}{dH} = K_p + \frac{A}{H^2} + \frac{B}{H^3} + \frac{C}{H^4} + \dots \quad (3)$$

$$I_m = aH_m + bH_m^2 + cH_m^3 + \dots \quad (4)$$

The coefficients of series (3) can be found from the theory of Akulov and Braun; the theoretical basis for the first two coefficients of series (4), Rayleigh's formula, is given by Preisach's theory [3], further refined and developed by Ye. I. Kondorskiy [4] and T. A. Yelkina [5].

As for the ascending and descending branches of the hysteresis loop, the following empirical formulas were obtained for weak fields by Rayleigh:

$$\begin{aligned} I^+ &= I_m - a(H_m - H) - \frac{b}{2}(H_m - H)^2, \\ I^- &= I_m - a(H_m - H) + \frac{b}{2}(H_m - H)^2, \end{aligned} \quad (5)$$

where I^+ is the descending branch, I^- the ascending branch, I_m and H_m the maximum coordinates of the loop, and I and H the variable coordinates of points moving on the descending and ascending branches, respectively.

Expanding the formulas into a Taylor's series in the difference $(H_m - H)$, we obtain the following more general expressions for the descending and ascending branches:

$$I^+ = \alpha_0 - \alpha_1(H_m - H) - \alpha_2(H_m - H)^2 - \alpha_3(H_m - H)^3 - \dots \quad (6)$$

$$I^- = \beta_0 - \beta_1(H_m - H) - \beta_2(H_m - H)^2 - \beta_3(H_m - H)^3 - \dots \quad (7)$$

We can establish relations among the coefficients by taking into account the fact that each point on the ascending branch corresponds to a point, symmetrical with respect to loop center, on the descending branch. This may be mathematically expressed by the condition that if we simultaneously change the sign of both I^+ and $(H_m - H)$ we obtain I^- :

$$I^- = \alpha_0 - \alpha_1(H_m - H) - \beta_2(H_m - H)^2 - \gamma(H_m - H)^3 - \dots \quad (8)$$

$$I^- = -\alpha_0 - \alpha_1(H_m - H) + \beta_2(H_m - H) - \gamma(H_m - H)^3 + \dots \quad (9)$$

It now remains for us to establish a connection between a, b, c, \dots on the one hand and $\alpha, \beta, \gamma, \dots$ on the other. Setting $H = H_m$ in (8) we obtain:

$$\alpha_0 = I_m.$$

Setting $H = -H_m$ we find:

$$I_m = -\alpha H_m - 2\beta H_m^2 - 4\gamma H_m^3 - \dots \quad (10)$$

- 2 -

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50X1-HUM

Comparing (10) and (4) term-wise, we obtain:

$$\alpha = a, \quad \beta = \frac{b}{2}, \quad \gamma = \frac{c}{4}, \quad \text{etc.}$$

Consequently, if the magnetization curve is given by the equation

$$I_m = aH_m + bH_m^2 + cH_m^3 + \dots,$$

then the descending and ascending branches of the hysteresis loop are expressed by the equations:

$$\begin{aligned} I^+ &= I_m - a(H_m - H) - \frac{b}{2}(H_m - H)^2 - \frac{c}{4}(H_m - H)^3 - \dots \\ I^- &= I_m - a(H_m - H) + \frac{b}{2}(H_m - H)^2 - \frac{c}{4}(H_m - H)^3 + \dots \end{aligned} \quad (11)$$

In the following we will consider the first three expansion terms, which will enable us to describe the form of the hysteresis loop in a considerably wider region than Rayleigh's.

Calculating the integral $\int_{-H_m}^{+H_m} (I^+ - I^-) \cdot dH$ on the basis of (11), we

find the hysteresis loss per cycle of magnetic reversal:

$$Q_h = \frac{4bH_m^3}{3} + 2cH_m^4 \quad (12)$$

Setting $H = 0$ in expression, we find the residual magnetism:

$$I_r = \frac{bH_m^2}{2} + \frac{3}{4}cH_m^3 \quad (13)$$

Through comparison of formulas (12) and (13) we find an extremely simple and important relation that links hysteresis loss with residual magnetism:

$$Q_h = \frac{8}{3}I_r H_m \quad (14)$$

NOTE: It is particularly important to note that this relation is correct for the calculation of not only the Rayleigh terms, but also the next term of the expansion, namely the third. If, however, we consider the following terms of the approximation, then this relation becomes violated.

To experimentally verify the relations derived, we employed Fe-Co specimens. Measurements were carried out on the Akulov-Bozort statistical magnetometer.

Figure 1 gives the theoretical I-H curve of magnetic intensity as calculated by formula (4) (a equals 8.7, b equals 1.33, c equals 0.3). Experimental data is represented by the points. In the figure, the Rayleigh curve as calculated by formula (1) is represented by the broken line. We see that the calculation of the third term makes it possible to encompass the significant part of the Stoletov region (the interval between the Rayleigh region and maximum permeability).

The same graph gives the curves of the ascending and descending branches of the hysteresis loop, as found by formula (11) and our measurements. If we take into account the fact that the whole family of curves involves only the three empirical parameters a, b, c, then we must recognize how extremely important it is to calculate the term supplementing Rayleigh's formulas.

- 3 -

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Figure 2 gives the hysteresis loss as found from formula (14) and our experimental data on the Fe-Co sample. The graph shows that calculation of the third term of the approximation makes it possible to encompass a region considerably wider than Rayleigh's.

Figure 3 compares the curve computed by formula (13) with our experimental data on residual magnetism. Analysis of this curve shows again how important it is to include the calculation of the next (third) term of the approximation.

In the literature, the opinion is sometimes expressed that Rayleigh's formulas are explicit, that they do not appear as the first two terms of a series expansion. Our investigations here obviously refute this opinion. In other words, no singular point which separates Rayleigh's region from later portions of Stoletov's curve exists in the case we studied.

It is important to note that formula (14) for hysteresis loss is accurate not only for the case where original magnetism is absent, but also for polarized magnetic materials; that is, when a ferromagnet is located in a pulsating magnetic field (in this case I_r is understood to be the residual magnetization of a partial cycle). The correctness of this follows from the fact that to obtain the specified relation, we had to calculate the symmetry of upper and lower hysteresis branches relative to loop center. For not too great amplitudes this symmetry definitely holds true. Figure 4 gives the curves describing hysteresis loss Q as a function of the maximum amplitude H_m of magnetic field strength for various submagnetic fields.

From our relations (11) and (4) follows the well-known formula:

$$I = I_m - 2f\left(\frac{H_m - H}{2}\right)$$

where $f(H_m)$ is the magnetization curve. This relation was obtained earlier by Ye. I. Kondorskiy (4) on the basis of a refinement of Preisach's model.

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50X1-HUM

[Appended figures follow.]

- 4 -

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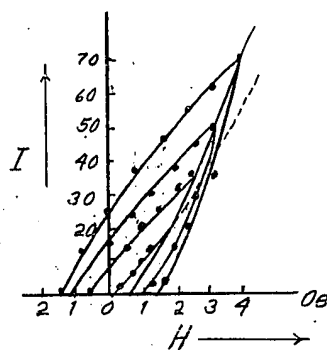


Figure 1

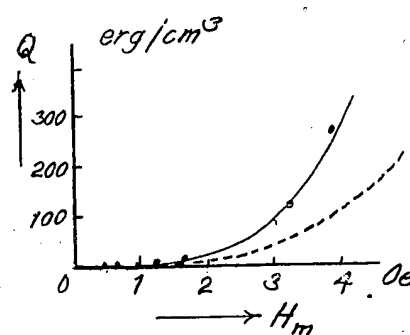


Figure 2

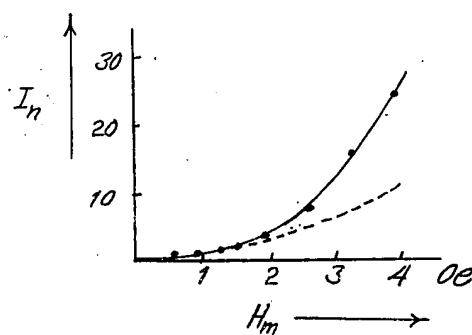


Figure 3

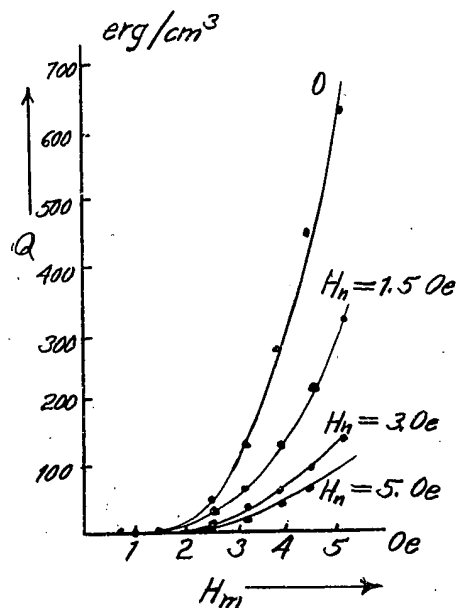


Figure 4

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- 5 -

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